Abrupt changes in the root-mean-square values of force coefficients
for an orbiting cylinder in uniform stream

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Abstract
A finite difference solution is presented for 2D laminar unsteady flow around a circular cylinder in orbital motion placed in a uniform stream at $Re=120-180$. Abrupt switches were found in the time-mean and root-mean-square values of lift, drag and base pressure coefficients under lock-in conditions. The phenomenon was investigated by computations at different values of Reynolds number, amplitude of in-line oscillation and ellipticity. Plotting against ellipticity suggests two states of vortex shedding exist.

Keywords: Circular cylinder; Ellipticity; Incompressible Navier-Stokes; Lift and drag coefficients; Lock-in; Orbital path; Reynolds number; rms values; Vortex shedding

1. Introduction
Oscillatory flow has been widely researched because of its practical importance, particularly in marine structures. An oscillating cylinder in uniform flow (e.g. Mahfouz and Badr (2000)), and for fluid at rest, an orbiting cylinder (see Teschauer et al. (2002)) and a simultaneously orbiting and rotating cylinder (see Stansby and Rainey (2001)) have been investigated, but research on an orbiting cylinder in uniform flow seems to have been ignored. However, flow around such bluff bodies moving with constant speed in waves, for instance, has practical applications, besides theoretical interest.

In this numerical investigation, an orbiting circular cylinder is placed in the uniform stream of an incompressible fluid at low Reynolds numbers. This revealed a previously unobserved phenomenon: the lift, drag and base pressure coefficients seem to switch abruptly from one state to another. This paper deals with the investigation of this phenomenon.

2. Governing equations and computational setup
To compute two-dimensional low-Reynolds number unsteady flow around a circular cylinder placed in a uniform stream and forced to follow an orbital path, a non-inertial system fixed to the cylinder is used. The non-dimensional Navier-Stokes equations for incompressible constant-property Newtonian fluid and the Poisson equation for pressure can be written as follows:

$$
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{1}{Re} \nabla^2 u - a_{ox},
$$

$$
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + \frac{1}{Re} \nabla^2 v - a_{oy},
$$

$$
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 2 \left[ \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \right] + \frac{\partial D}{\partial t}.
$$

At each time step all equations are solved and the continuity equation is satisfied. Standard notations are used here, $a_{ox}, a_{oy}$ are the components of cylinder acceleration, $D$ is dilation, and $Re$ is the Reynolds number defined as $Re = Ud / \nu$ where $d$ is the diameter of the cylinder, $U$ is the upstream velocity and $\nu$ is the kinematic viscosity.

No-slip boundary condition is used on the cylinder surface for the velocity and a Neumann-type condition is used for pressure $p$. Potential flow distribution is assumed far from the cylinder. Boundary conditions can only be imposed accurately when using boundary fitted coordinates, and therefore the physical domain is transformed into an equidistant computational domain. The transformed equations are solved by using finite difference method. For further details see Baranyi and Shirakashi (1999).

Figure 1 shows the flow arrangement. The orbital motion of the cylinder is created by the superposition of two forced oscillations with identical frequencies. The motion of the center of the cylinder with unit diameter is specified as follows:

$$
x_a(t) = A_x \cos(2\pi f_x t);
$$

$$
y_a(t) = A_y \sin(2\pi f_y t)
$$

where $A_x, A_y$ and $f_x, f_y$ are the dimensionless amplitudes and frequencies of oscillations in $x$ and $y$ directions, respectively. Here $f_x = f_y$ which for nonzero $A_x, A_y$ amplitudes gives an ellipse, shown in the
dotted line in Figure 1. \( A_y \) alone yields pure in-line oscillation, and then as \( A_y \) is increased, the ellipticity \( E = A_y / A_x \) increases to yield a full circle at \( E = 1 \).

During each set of computations, \( Re \) and \( A_x \) are fixed and \( f_x \) and \( f_y \) are kept constant at 85\% of the frequency of vortex shedding from a stationary cylinder at that \( Re \). This value was chosen to ensure lock-in (synchronisation of frequencies of vortex shedding and cylinder oscillation) at moderate oscillation amplitudes.

### 3. Computational results and discussion

An interesting phenomenon was found under lock-in condition when looking at the time-mean value (TMV; also denoted by overbar) and root-mean-square (\( rms \)) values of lift (\( C_L \)), drag (\( C_D \)) and base pressure (\( C_{pb} \)) coefficients for an orbiting cylinder in a uniform flow. Abrupt jumps were found when these values were plotted against ellipticity \( E \) with \( Re \) and \( A_x \) kept constant (see Baranyi (2003) and Baranyi (2004)). A typical example for the TMV of lift coefficient \( C_L \) is shown in Figure 2 where three sudden jumps in the curve can be seen. Both upper and lower curves are almost straight lines and in general their slopes are almost identical. Two different states were found on the curve of \( C_L \) versus \( E \), one with greater lift, the other with smaller. Both show an approximately linear decrease with increasing \( E \), and the difference between the \( C_L \) values belonging to the two states is approximately constant. It was shown in Baranyi (2003) that the time histories of \( C_L \) before and after the jumps are substantially different.

In the earlier paper of the author (see Baranyi (2003)) it was found that the clearest example of abrupt jumps was in \( C_L \) so the author’s next study (see Baranyi (2004)) focused on the investigation of the effects of \( Re \) and \( A_x \) on this quantity. Later, however, it was found that for values of \( A_y \) larger than investigated in Baranyi (2003), the jumps are also pronounced in the TMV and \( rms \) values of other force coefficients. That is why this paper focuses on the investigation of \( C_{pb} \), \( C_{Lrms} \), \( C_{Drms} \) and \( C_{pbms} \). Computations were carried out for several \( Re \) values: 120, 140, 160 and 180 at \( A_x = 0.4 \) and 0.5. Two typical examples are shown in Figures 3 and 4. Figure 3 shows \( C_{L rms} \) values versus \( E \) for \( Re=160 \) and \( A_x = 0.4 \) while Figure 4 demonstrates the variation of \( C_{pb} \) with \( E \) at \( Re=140 \) and \( A_x = 0.5 \). The jumps can be seen clearly in both figures. These also support the idea that there are two states, or two solutions and the solution jumps from one state to the other and back. When comparing Figure 2 with Figures 3 and 4 the main difference between the curves representing the two states is obvious: the curves intersect each other at \( E = 0 \) and the difference between the upper and lower curves increases with increasing \( E \) in Figures 3 and 4, while the two curves are almost parallel with each other in Figure 2. Basically similar results were obtained for \( C_{pb} \), and for \( C_{L rms} \), \( C_{Drms} \) and \( C_{pbms} \). So by increasing the amplitude of transverse oscillation \( A_y \) (that is, \( E \)) the difference between the TMV or \( rms \) values belonging to the two states is increasing for these quantities, or rather this is true for the small \( E \) values. As can be seen in Figure 4 the distance between the curves representing the two states starts to decrease at around \( E = 0.8 \) for that particular case.
Although computations were carried out for both $A_x=0.4$ and 0.5 for the Re numbers mentioned earlier, here only the $A_x=0.4$ cases will be shown. Although $A_x$ has some effects on the curve the two sets of curves are quite similar. Figures 5 shows $C_{pb}$ versus $E$ for the four Reynolds numbers specified earlier. Figures 6-8 show $C_{Lrms}$, $C_{Drms}$ and $C_{pbrms}$, respectively. The effect of Reynolds number can clearly be seen in these figures. Curves representing the two states remain similar in shape for different Reynolds numbers but by increasing Re they tend to shift upward. Another interesting point to mention is that while for stationary cylinder $C_{Drms}$ and $C_{pbrms}$ are very small, for oscillating cylinders the rms values of these quantities become quite large; comparable in magnitude to $C_{Lrms}$. For stationary cylinder the present author found that in the $Re=120-180$ domain $C_{Drms}=0.01-0.026$ and $C_{pbrms}=0.029-0.07$. By comparing these values with the values in Figures 7 and 8 it can be seen that due to the orbital motion the amplitude of oscillation was increased substantially. $C_{Lrms}$ were also increased in this $Re$ number domain from $C_{Lrms}=0.285-0.436$ for the stationary cylinder to the values shown in Figure 6 but the relative increase in this case is smaller than for the other two rms values.

Although the possibility that the jumps are due to the code itself should not be totally ruled out, it seems highly unlikely, as the large amount of computational results are consistent and the code has proven to be very reliable at these Reynolds numbers against experimental values for stationary cylinders (see Baranyi and Shirakashi (1999)).

The possibility of a relation to 3D instability which occurs in stationary cylinders around $Re=188.5$ (see Barkley and Henderson (1996)) was considered, but this phenomenon occurs at lower Re and it has been shown that oscillation increases the two-dimensionality of the flow (see Poncet (2002)).
explanation that can be offered at this time is that the structure of vortices changes, similarly to those shown in Williamson and Roshko (1988) for transverse oscillation. It seems that two solutions co-exist, with one solution nearer to the initial state, and when this destabilizes, the solution jumps to the second solution and remains until that becomes unstable. This appears to be a possible case of bifurcation. Experimental investigations and/or stability analysis are needed to confirm this hypothesis.

Cylinder in orbital motion; $C_{D_{rms}}, A_x=0.4$

Figure 7

$R_m$s values of drag coefficient vs. ellipticity

Cylinder in orbital motion; $C_{p_{brms}}, A_x=0.4$

Figure 8

Time-mean base pressure coefficient vs. ellipticity

4. Conclusions
The main findings of this study were:
- The shape of the time-history signal of the base pressure coefficient and the $r_m$s values of lift, drag and base pressure coefficients changed drastically at critical ellipticity values, resulting in abrupt jumps in the TMV and $r_m$s values of the force coefficients mentioned.
- When plotted against ellipticity $E$, two different states co-exist.
- The two curves representing the two states intersect each other at $E=0$ and then the distance between the two curves increases with $E$.
- An increase in Reynolds number tends to shift the two curves upward.
- The $r_m$s values of drag and base pressure coefficients for orbiting cylinders increase substantially compared to the values for stationary cylinders.

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References