Fuzzy Rule Interpolation in Practice

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Abstract — There are relatively few Fuzzy Rule Interpolation (FRI) techniques can be found among the practical fuzzy rule based applications. On one hand the FRI methods are not widely known, and many of them have limitations from practical application point of view, e.g. can be applied only in one dimensional case, or defined based on the two closest surrounding rules of the actual observation. On the other hand enabling the application of sparse rule bases the FRI methods can dramatically simplify the way of fuzzy rule base creation, since FRI methods can provide reasonable (interpolated) conclusions even if none of the existing rules fires under the current observation. These methods can save the expert from dealing with derivable rules and help to concentrate on cardinal actions only and hence simplify the rule base creation itself. Thus, compared to the classical fuzzy CRI, the number of the fuzzy rules needed to be handled during the design process, could be dramatically reduced. In this paper, among the brief structure of several FRI methods, a simple and quick FRI method “FIVE” will be introduced in more details, and for demonstrating the benefits of the interpolation-based fuzzy reasoning as systematic approach, the construction of a fuzzy rule base through a simple example will be also discussed.

Index Terms — Fuzzy Rule Interpolation (FRI), Interpolation-based fuzzy reasoning, FRI applications

I. INTRODUCTION

Since the classical fuzzy reasoning methods (e.g. compositional rule of inference (CRI)) are demanding complete rule bases, the classical rule base construction claims a special care of filling all the possible rules. In case if there are some rules missing (the rule base is “sparse”), observations may exist which hit no rule in the rule base and therefore no conclusion is obtained. Having no conclusion in a fuzzy control structure is hard to explain. E.g. one solution could be to keep the last real conclusion instead of the missing one, but applying historical data automatically to fill undeliberately missing rules could cause unpredictable side effects. Another solution for the same problem is the application of the fuzzy rule interpolation (FRI) methods, where the derivable rules are deliberately missing, since FRI methods can provide reasonable (interpolated) conclusions even if none of the existing rules fires under the current observation. The rule base of an FRI controller is not necessarily complete; hence it could contain the most significant fuzzy rules only without risking the chance of having no conclusion for some of the observations. On the other hand most of the FRI methods are sharing the burden of high computational demand, e.g. the task of searching for the two closest surrounding rules to the observation, and calculating the conclusion at least in some characteristic α-cuts. Moreover in some methods the interpretability of the fuzzy conclusion gained is also not straightforward [7]. There have been a lot of efforts to rectify the interpretability of the interpolated fuzzy conclusion [17]. In [1] Baranyi et al. give a comprehensive overview of the recent existing FRI methods. Beyond these problems, some of the FRI methods are originally defined for one dimensional input space, and need special extension for the multidimensional case (e.g. [2]-[3]). In [22] Wong et al. gave a comparative overview of the recent multidimensional input space capable FRI methods. In [2] Jenei introduced a way for axiomatic treatment of the FRI methods. In [14]Perfilieva studies the solvability of fuzzy relation equations as the solvability of interpolating and approximating fuzzy functions with respect to a given set of fuzzy rules (e.g. fuzzy data as ordered pairs of fuzzy sets). The high computational demand, mainly the search for the two closest surrounding rules to an arbitrary observation in the multidimensional antecedent space makes many of these methods hardly suitable for real-time applications. Some FRI methods, e.g. the method introduced by Jenei et al. in [3], eliminate the search for the two closest surrounding rules by taking all the rules into consideration, and therefore speed up the reasoning process. On the other hand, keeping the goal of constructing fuzzy conclusion, and not simply speeding up the reasoning process, they still require some additional (or repeated) computational steps for the elements of the level set (or at least some relevant α levels). A rather different application oriented aspect of the FRI emerges in the concept of “FIVE” (Fuzzy Interpolation based on Vague Environment). In the followings, among the brief structure of several FRI methods, the “FIVE” will be introduced in more details.

II. A BRIEF OVERVIEW OF SEVERAL FRI TECHNIQUES

One of the first FRI techniques was published by Kóczy and Hirota [5]. It is usually referred as KH method. It is applicable to convex and normal fuzzy (CNF) sets. It determines the conclusion by its α-cuts in such a way that the ratio of distances between the conclusion and the consequents.
should be identical with the ones between the observation and the antecedents for all important \( \alpha \)-cuts. The applied formula 
\[
d(A^*, A_x) : d(A^*, A_y) = d(B^*, B_x) : d(B^*, B_y)\]
can be solved for \( B^* \) for relevant \( \alpha \)-cuts after decomposition.

It is shown in, e.g. in [7], [8] that the conclusion of the KH method is not always directly interpretable as fuzzy set. This drawback motivated many alternative solutions. A modification was proposed by Vass, Kalmár and Kóczy [20] (VKK method), where the conclusion is computed based on the distance of the centre points and the widths of the \( \alpha \)-cuts, instead of lower and upper distances. VKK method decreases the applicability limit of KH method, but does not eliminate it completely. The technique cannot be applied if any of the antecedent sets is singleton (the width of the antecedent’s support must be nonzero). In spite of the disadvantages, KH is popular because its simplicity that infers its advantageous complexity properties. It was generalized in several ways. Among them the stabilized KH interpolator is emerged, as it is proved to hold the universal approximation property [19], [16]. This method takes into account all flaking rules of an observation in the calculation of the conclusion in extent to the inverse of the distance of antecedents and observation. The universal approximation property holds if the distance function is raised to the power of the input’s dimension.

Another modification of KH is the modified alpha-cut based interpolation (MACI) method [17], which alleviates completely the abnormality problem. MACI’s main idea is the following: it transforms fuzzy sets of the input and output universes to such a space where abnormality is excluded, then computes the conclusion there, which is finally transformed back to the original space. MACI uses vector representation of fuzzy sets and originally applicable to CNF sets [23]. These latter conditions (CNF sets) can be relaxed, but it increases the computational need of the method considerably [18]. MACI is one of the most applied FRI methods [22], since it preserves advantageous computational and approximate nature of KH, while it excludes its abnormality.

Another fuzzy interpolation technique was proposed by Kóczy et al. [6]. It is called conservation of “relative fuzziness” (CRF) method, which notion means that the left (right) fuzziness of the approximated conclusion in proportion to the flanking fuzziness of the neighboring consequent should be the same as the (left) right fuzziness of the observation in proportion to the flanking fuzziness of the neighboring antecedent. The technique is applicable to CNF sets.

An improved fuzzy interpolation technique for multidimensional input spaces (IMUL) was proposed in [21], and described in details in [22]. IMUL applies a combination of CRF and MACI methods, and mixes advantages of both. The core of the conclusion is determined by MACI method, while its flanks by CRF. The main advantages of this method are its applicability for multi-dimensional problems and its relative simplicity.

Conceptually different approaches were proposed by Baranyi et al [1] based on the relation and on the semantic and inter-relational features of the fuzzy sets. The family of these methods applies “General Methodology” (GM); this notation also reflects to the feature that these methods are able to process arbitrary shaped fuzzy sets. The basic concept is to calculate the reference point of the conclusion based on the ratio of the distances between the reference points of the observation and the antecedents. Then, a single rule reasoning method (revision function) is applied to determine the final fuzzy conclusion based on the similarity of the fuzzy observation and an “interpolated” observation.

III. A SIMPLE AND QUICK FRI METHOD: “FIVE”

A rather different application oriented aspect of the fuzzy rule interpolation emerges in the concept of FIVE. The fuzzy reasoning method “FIVE” (Fuzzy Interpolation based on Vague Environment, originally introduced in [9], [10] and [11]) was developed to fit the speed requirements of direct fuzzy control, where the conclusions of the fuzzy controller are applied directly as control actions in a real-time system.

![Fig. 1: Interpolation of two fuzzy rules (Ri: Ai→Bi), by the Shepard operator based FIVE, and for comparison the min-max CRI with COG defuzzification.](image)

The main idea of the FIVE is based on the fact that most of the control applications serve crisp observations and requires crisp conclusions from the controller. Adopting the idea of the vague environment (VE) [4], FIVE can handle the antecedent and consequent fuzzy partitions of the fuzzy rule base by scaling functions [4] and therefore turn the fuzzy interpolation to crisp interpolation.

The idea of a VE is based on the similarity (in other words: indistinguishability) of the considered elements. In VE the fuzzy membership function \( \mu_a(x) \) is indicating level of similarity of \( x \) to a specific element \( a \) that is a representative or prototypical element of the fuzzy set \( \mu_a(x) \), or, equivalently, as the degree to which \( x \) is indistinguishable from \( a \) [4]. Therefore the \( \alpha \)-cuts of the fuzzy set \( \mu_a(x) \) are the sets which contain the elements that are \((1-\alpha)\)-
indistinguishable from \( a \). Two values in a \( \text{VE} \) are e-distinguishable if their distance is greater than \( \varepsilon \). The distances in a \( \text{VE} \) are weighted distances. The weighting factor or function is called scaling function (factor) \cite{4}. If \( \text{VE} \) of a fuzzy partition (the scaling function or at least the approximate scaling function \cite{9,11}) exists, the member sets of the fuzzy partition can be characterized by points in that \( \text{VE} \) (see e.g. scaling function \( s \) on fig. 1). Therefore any crisp interpolation, extrapolation, or regression method can be adapted very simply for FRI \cite{9,11}. Because of its simple multidimensional applicability, in FIVE the Shepard operator based interpolation (first introduced in \cite{15}) is adapted (see e.g. fig. 1).

The code of the FIVE FRI together with other FRI methods as a freely available FRI Toolbox can be downloaded at \cite{24}.

IV. FRI RULE BASE DESIGN EXAMPLE

The application example introduced in this paper for demonstrating the benefits of the interpolation-based fuzzy reasoning as systematic approach, is the fuzzy rule base construction of an automated guided vehicle (AGV) steering control \cite{12,13}. In the example, the steering control has two goals, the path tracking (to follow a guide path) and the collision avoidance. The guide path is usually a painted marking or an active guide-wire on the floor. The guiding system senses the position of the guide path by special sensors (guide zone) tuned for the guide path. The goal of the steering control is to follow the guide path by the guide zone with minimal path tracking error on the whole path (fig. 2).

\[ \delta = \begin{array}{c|ccc|ccc|ccc} & NL & NM & Z & PS & NS & PL \\ \hline \varepsilon = & NL & PS & Z & NS & NL & PL \\ NM & PL & PS & Z & NL & PL & Z \\ Z & PL & PL & PS & Z & NL & PL \\ PS & PL & PL & Z & NL & PL & Z \\ PL & PS & PS & Z & NS & PL & NS \\ \end{array} \]

\[ \text{TABLE I} \]

A. Path tracking, complete rule base

The design of the steering control rule base can be divided into two main steps. First the path tracking rule base needed to be elaborated and then it can be extended by the rules of collision avoidance. The simplest way of defining the fuzzy rules is based on studying the operator’s control actions in relevant situations. These control actions could form the later rule base. The basic idea of the path tracking strategy is very simple: keep the driving centre \( K \) of the AGV as close as it is possible to the guide path, and than simply turn the AGV into the new direction. This strategy needs two observations: the measured distance between the guide path and the guide point \( (e_c) \), and the estimated distance between the guide path and the driving centre \( (\delta) \) (see fig. 2 for the notation). Based on these observations, as a conclusion, the level of steering \( \text{V}_d = \text{V}_L - \text{V}_R \) needed to be calculated. Collecting the operator’s control actions, the path tracking strategy can be characterized by five relevant situations, collected on fig. 3.

\[ \text{Fig. 3. Relevant control actions (Vd: steering) characterizing the path tracking strategy (see Fig. 2. for the notation).} \]

Having the relevant control actions and the linguistic term fuzzy sets (fuzzy partitions) of the two antecedent and one consequent universes, the fuzzy rule base can be simply constructed. The \( l \)-th rule of the rule base has the form:

\[ \text{R}_v^{(l)} : \text{If } e_c = A_{1l} \text{ And } \delta = A_{2l} \text{ Then } \text{V}_d = B_1. \]

Let us have the linguistic term fuzzy partitions built up five fuzzy sets, namely: negative large (NL), negative middle (NM), zero (Z), positive middle (PM), positive large (PL) for the two antecedents universes \( (e_c, \delta) \), and negative large (NL), negative small (NS), zero (Z), positive small (PS), positive large (PL) for the consequent universe \( (\text{V}_d) \). Building a complete fuzzy rule base first, according to the antecedent terms, we have to set up an antecedent grid of all possible fuzzy rules, and then fill it with the corresponding rule consequents (see Table I). First we can fill the rule consequents already known as relevant situations from the knowledge acquisition phase (noted by underline on Table I.), than to make the rule base complete, the “filling” rules too.

In most cases, the “filling” rules have the only task to get “smooth transient” between the relevant rules. Selecting a fuzzy reasoning method, e.g. the max-min composition, and center of gravity defuzzification, the control surface of the steering can be directly calculated (see fig. 4).

\[ \begin{array}{c|c|c|c|c|c|c|c|c} & NL & NM & Z & PS & NS & PL \\ \hline \text{TABLE I} \]

B. Path tracking, sparse rule base

The design of the path tracking steering control rule base for FRI is very similar to the complete rule base situation. The
main difference is the lack of the “filling” rules. The rule base contains the rules of the relevant situations, known from the knowledge acquisition phase, only (see Table II).

Introducing single antecedent rules, rules which have the same conclusion independently from some of the antecedents can be merged to single rules i.e. according to our example the rule base of Table II can be simplified to Table III.

Selecting a fuzzy reasoning method suitable for sparse rule bases, i.e. in our case the “FIVE” FRI, introduced in Section III, after constructing the scaling functions of the antecedent and consequent universes based on their fuzzy partitions (see fig. 6, fig. 7, fig. 8), the control surface of the steering can be directly calculated (see fig. 5).

Fig. 4. Control surface of the path tracking steering strategy, max-min CRI, centre of gravity defuzzification, 25 rules, complete rule base (Table I.)

TABLE II

<table>
<thead>
<tr>
<th>RVd :</th>
<th>e_v</th>
<th>δ</th>
<th>Vd</th>
</tr>
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<tbody>
<tr>
<td>NL</td>
<td>PL</td>
<td>NL</td>
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<tr>
<td>NM</td>
<td>PL</td>
<td>PS</td>
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<td>Z</td>
<td>PL</td>
<td>Z</td>
<td>NL</td>
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<tr>
<td>PM</td>
<td>PL</td>
<td>NS</td>
<td>NL</td>
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<td>PL</td>
<td>PL</td>
<td>NL</td>
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</table>

Fig. 5. Control surface of the path tracking steering strategy, “FIVE” FRI, 8 rules, sparse rule base, according to Table III.

TABLE III

<table>
<thead>
<tr>
<th>RVd :</th>
<th>e_v</th>
<th>δ</th>
<th>Vd</th>
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<tbody>
<tr>
<td>Rule 1:</td>
<td>NL</td>
<td>PL</td>
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<tr>
<td>Rule 2:</td>
<td>PL</td>
<td>NL</td>
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<td>Rule 3:</td>
<td>NM</td>
<td>Z</td>
<td>PL</td>
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<tr>
<td>Rule 4:</td>
<td>PM</td>
<td>Z</td>
<td>NL</td>
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<td>Rule 5:</td>
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<td>Rule 6:</td>
<td>PM</td>
<td>NM</td>
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</tr>
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<td>Rule 7:</td>
<td>Z</td>
<td>PM</td>
<td>NS</td>
</tr>
<tr>
<td>Rule 8:</td>
<td>Z</td>
<td>NM</td>
<td>PS</td>
</tr>
</tbody>
</table>

C. Path tracking, and collision avoidance, sparse rule base

For extending the path tracking rule base by the rules of collision avoidance, first the new observations required for detecting collision situations are needed to be defined.

In our example collision avoidance strategy, two different collision situations, the frontal and the side collision are distinguished. For the obstacle sensor configuration, having the precondition of motionless obstacles, it is sufficient to have three ultrasonic distance sensors at the front of the AGV, one in the middle (U_M) and one-one on both sides (U_L, U_R) (see fig. 2)) to approximate both the collision conditions [13]. Since in case of motionless obstacles, the obstacle distance measurements of the near past can be used for scanning the boundaries of the obstacles. Collecting the previous measurements of the left and right obstacle sensors and the corresponding positions of the AGV (measured by the motion sensors on the wheels), the boundaries of the obstacles can be
approximated [13].

Hence for avoiding frontal collision situations the distances measured by the left middle and right ultrasonic sensors (\(R_L, R_M, R_R\)) can be applied as additional observations for the steering control, and for the side collision situations based on the approximated boundaries of the obstacles, the approximated maximal left and right turning angle without side collision (\(\alpha_{ML}, \alpha_{MR}\)) can be applied as additional observations. Together with the two original observations of the steering control it takes seven observations, therefore the \(i^{th}\) rule of the rule base will have the following form:

\[
R_{V_d(i)}: \text{If } e_v = A_{1,i} \text{ And } \delta = A_{2,i} \text{ And } R_L = A_{3,i} \text{ And } R_R = A_{4,i} \text{ And } R_M = A_{5,i} \text{ And } \alpha_{ML} = A_{6,i} \text{ And } \alpha_{MR} = A_{7,i} \text{ Then } V_d = B_i.
\]

In this example the main goal of the Path tracking and collision avoidance strategy is the path tracking (to follow a guide path) and as a sub goal, a kind of restricted (limited) collision avoidance. Here the restricted collision avoidance means: “avoiding obstacles without risking the chance of loosing the guide path”.

The extension of the path tracking rule base (Table III) with collision avoidance can be done in the similar way, as the original rule base was designed. Having FRI, it is enough to concentrate on the relevant situations only. First we have to define the linguistic term fuzzy partitions of the new observations (namely for: \(R_L, R_M, R_R, \alpha_{ML}, \alpha_{MR}\), and then by collecting the control actions of the relevant situations, setting up the new rules. For the new antecedents universes it is enough to have two fuzzy sets in the linguistic term fuzzy partitions, namely large (\(L\)), and small (\(S\)), as we are mainly interested in the existence of the collision situation (small value (\(S\)): collision situation, large value (\(L\)): no collision situation). The rest of the antecedent universes and the consequent universe may remain unchanged.

At the first step of introducing collision avoidance in the path tracking rule base, we have simply extend the original rule base (Table III) by simply permitting the control actions if there is no collision situation (\(L\)) concerned (see Table IV). The rules we get such a way (Table IV), has the same effect as the original path tracking rules (Table III), if there is no obstacle, but they are loosing their influence if a corresponding collision situation appears.

<table>
<thead>
<tr>
<th>(R_{V_d})</th>
<th>(e_v)</th>
<th>(\delta)</th>
<th>(R_L)</th>
<th>(R_R)</th>
<th>(R_M)</th>
<th>(\alpha_{ML})</th>
<th>(\alpha_{MR})</th>
<th>(V_d)</th>
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<tbody>
<tr>
<td>Rule 1</td>
<td>NL</td>
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<tr>
<td>Rule 2</td>
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<tr>
<td>Rule 3</td>
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<td>LPL</td>
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<tr>
<td>Rule 4</td>
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<td>Rule 5</td>
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<tr>
<td>Rule 6</td>
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<td>Rule 7</td>
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<td>Rule 8</td>
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The next step is handling the collision situations. Keeping the structure of the original relevant path tracking rules, we have to add rules which gaining influence, when an obstacle is approaching (\(S\)). In our simple example, we need only four additional rules. Two rules (9, 10 of Table V) are required for modifying the consequence of the original path tracking rules (7, 8 of Table V) in collision situation, and two other rules (11, 12 of Table V) for handling the frontal collision situation.

<table>
<thead>
<tr>
<th>(R_{V_d})</th>
<th>(e_v)</th>
<th>(\delta)</th>
<th>(R_L)</th>
<th>(R_R)</th>
<th>(R_M)</th>
<th>(\alpha_{ML})</th>
<th>(\alpha_{MR})</th>
<th>(V_d)</th>
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<tr>
<td>Rule 9</td>
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<td>Rule 10</td>
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<td>Rule 12</td>
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The rule base (Table V) we got, is slightly differs from our original goal. Its main task is the collision avoidance and the path tracking remains the sub goal only. To keep the original goal the Path tracking and, restricted collision avoidance, “avoiding obstacles without risking the chance of loosing the guide path”, the rule base of Table V needs a slight modification. The main rules of path tracking (1, 2 of Table V) needed to be kept even if an obstacle is approaching, by removing the antecedents related to obstacles (\(S\)). Therefore for the final sparse rule base of the Path tracking and, restricted collision avoidance strategy we get Table VI.

The next step is constructing the scaling functions of the new antecedent universes based on their fuzzy partitions. The last step of the steering control design is the fine tuning of the whole system by parameter optimizing the antecedent and consequent scaling functions by a gradient free optimization method. In our case a simple hill climbing method was applied to optimize the docking distance of the AGV in case of a given obstacle configuration. Fig. 9 introduces two simulated run of the optimized steering control, one for the obstacle free, and one for the obstacle disturbed situation.
The main goal of this paper was to introduce a practical, application oriented view of Fuzzy Rule Interpolation (FRI) techniques. Applying FRI methods, and hence sparse rule bases can dramatically simplify the way of fuzzy rule base creation. FRI methods can save the expert from dealing with derivable rules and therefore help to reduce the number of the fuzzy rules needed to be handled considerably. In the example “Path tracking and, restricted collision avoidance strategy” introduced in this paper, the steering control sparse fuzzy rule base was built upon 12 rules only. In case of classical FLC e.g. max-min CRI, and complete rule base, fuzzy rule base was build upon 12 rules only. Having the same antecedent fuzzy partitions, the steering control should contain $5^2 + 2^1 = 57$ rules.

The code of the example application of this paper together with other FRI applications, and a freely available FRI Toolbox (a collection of Matlab functions implementing FRI techniques, under GNU General Public License) can be downloaded from [24].

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