

# Numerical simulation of oscillatory flow past and heat transfer from a cylinder

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**Abstract**—Two-dimensional low-Reynolds number flow ( $Re=160$ ) about a transversely oscillating heated circular cylinder placed in a uniform stream is investigated numerically using the commercial software package Ansys Fluent. The flow is simulated in a frame fixed to the cylinder and with an oscillating cross flow. The effect of oscillation amplitude on the force coefficients and heat transfer characteristics are investigated in the locked-in domain for four different temperature ratios. Comparisons include the mechanical energy transfer, time-mean and root-mean-square values of the lift and drag coefficients and Nusselt number.

**Keywords**— Heat transfer, Lift and drag coefficients, Mechanical energy transfer, Nusselt number, Transverse oscillation

## I. INTRODUCTION

THE flow around a circular cylinder placed in a uniform stream is an important engineering problem in fluid mechanics. Its physical and engineering applications have attracted the attention of engineers and scientists for over a century, leading to many theoretical, experimental and numerical investigations [1]-[3]. In addition, heat transfer becomes an issue in applications such as chimneys, tube bundles of heat exchangers, and hot wire anemometers.

For flows over a heated cylinder the fluid properties such as viscosity, density and thermal conductivity vary with the temperature. Consequently the thermodynamic properties in the system of the governing equations also become temperature dependent. This has a significant effect on the flow characteristics, making the flow phenomena much more complex than in the isothermal case. The vortex shedding from a cylinder can be reduced or even completely suppressed by increasing the cylinder temperature [4]-[6].

For isothermal case the flow around a transversely oscillating circular cylinder has been studied extensively with focus on among others the shedding regime, phase angle between body displacement and transverse force [3], [7]. Heat

transfer has been investigated for a transversely oscillating cylinder in a uniform stream at low Reynolds numbers in [8], [9]. To the best knowledge of the authors, the effect of surface temperature on the mechanical energy transfer between the fluid and cylinder has not attracted much attention so far.

The objective of the present work is therefore to investigate the effect of cylinder surface temperature on heat transfer from a heated circular cylinder placed in a uniform stream and oscillating in transverse direction, and on force coefficients and mechanical energy transfer between fluid and cylinder.

## II. NUMERICAL METHOD

The computational domain is characterized by two concentric circles: the inner represents the cylinder surface with diameter  $d$ , the outer the far field with diameter  $d_\infty$  (see Fig. 1). The origin of the coordinate system is in the center of the cylinder and the positive  $x$ -axis is directed downstream.

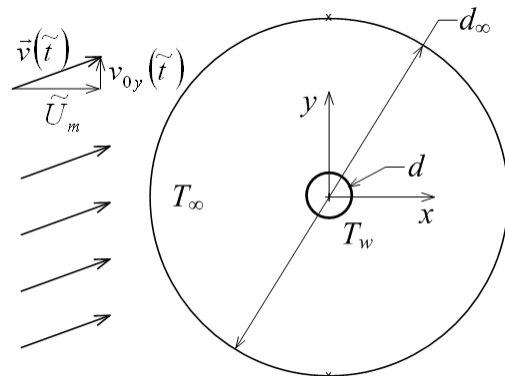


Fig. 1 Computational domain

The flowing fluid is air, considered to be incompressible. Far from the cylinder constant absolute temperature  $\tilde{T}_\infty$  (K) is prescribed, while the cylinder surface is kept at constant temperature  $\tilde{T}_w$  (K). Temperature ratio is defined as

$$T^* = \tilde{T}_w / \tilde{T}_\infty. \quad (1)$$

Four temperature ratios are investigated: 1.0, 1.1, 1.2 and 1.5. Especially for the larger  $T^*$  values the temperature difference between the fluid and the cylinder surface is large enough to

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influence the fluid properties, therefore the dependence of fluid properties on the temperature is taken into account.

At the inlet the time-dependent free stream undisturbed velocity is defined as the superposition of uniform free stream  $\tilde{U}_m$  (m/s) and an oscillatory flow in transverse direction

$$\mathbf{v}(\tilde{t}) = \tilde{U}_m \mathbf{i} + v_{0y} \mathbf{j} = \tilde{U}_m \mathbf{i} - 2p \tilde{f} \tilde{A}_y \cos(2p \tilde{f} \tilde{t}) \mathbf{j}, \quad (2)$$

where  $v_{0y}$  is the time dependent fluctuating velocity,  $\tilde{f}$  (1/s) is the frequency of oscillation,  $\tilde{A}_y$  (m) is the oscillation amplitude for transverse motion,  $\tilde{t}$  (s) is the time and  $\mathbf{i}, \mathbf{j}$  are the unit vectors in  $x$  and  $y$  directions (see Fig. 1), respectively. By introducing length and velocity scales all quantities can be non-dimensionalized. The non-dimensional time  $t$ , frequency of oscillation  $f$  and oscillation amplitude  $A_y$  can be written

$$t = \tilde{t} \tilde{U}_m / d, \quad f = \tilde{f} d / \tilde{U}_m, \quad A_y = \tilde{A}_y / d. \quad (3)$$

The dimensionless oscillation frequency was set at  $f=0.8St_0=0.14912$ , where  $St_0$  is the non-dimensional vortex shedding frequency, or Strouhal number, for a stationary cylinder at  $Re=160$ . This frequency ratio value ensures that lock-in condition can be reached at moderate amplitude values. Lock-in or synchronization happens when the vortex shedding frequency synchronizes with the frequency of cylinder motion. Here only locked-in cases are considered.

The governing equations and boundary conditions are solved by the commercial software package Ansys Fluent, based on the finite volume method (FVM). The accuracy of the computed results depends on the resolution (number, shape and distribution of cells), the time step, the size and shape of the computational domain. In [10] the effect of domain size, the mesh, and the time step is investigated on the solution of a low-Reynolds number flow around a stationary circular cylinder. The influence of computation domain size was investigated further in [11], and computational results were compared with those of several studies, finding very good agreement. In this study a mesh with domain size  $d_\infty/d=180$  and  $360 \times 298$  (azimuthal  $\times$  radial) cells compromises between computational cost and accuracy. In the physical domain logarithmically spaced radial cells are used, providing a fine grid near the cylinder wall and a coarse grid in the far field. The minimal dimensionless mesh size in radial direction is 0.00875, and the dimensionless time step of  $\Delta t=0.025$  is used.

### III. COMPUTATIONAL RESULTS

#### A. Lift and drag coefficient

The accuracy of the numerical results is compared by means of integral quantities such as lift  $C_L$  and drag  $C_D$  coefficients. The lift and drag coefficients are defined as

$$C_L = \frac{2F_L}{\rho \tilde{U}_m^2 d}, \quad C_D = \frac{2F_D}{\rho \tilde{U}_m^2 d}, \quad (4)$$

where  $\rho$  is the fluid density,  $d$  is the cylinder diameter,  $F_L$  and  $F_D$  is the lift and drag force per unit length of the cylinder. The time-mean and root-mean-square (rms) values of the lift and drag coefficients are calculated as

$$C_{TM} = \frac{1}{nP} \int_{t_1}^{t_1+nP} C(t) dt, \quad C_{rms} = \sqrt{\frac{1}{nP} \int_{t_1}^{t_1+nP} [C(t) - C_{TM}]^2 dt}, \quad (5)$$

where  $P$  is a period of a vortex shedding,  $n$  is the number of periods and  $t_1$  is the starting point of calculation. Coefficient  $C$  in (5) stands for drag and lift coefficients. Unless otherwise indicated, the lift and drag coefficients shown in this study do not contain inertial forces originating from the system fixed to the accelerating cylinder. Coefficients obtained by removing the inertial forces are often termed ‘fixed body’ coefficients [7]. The relationship between the two sets of coefficients can be written as

$$C_L = C_{L_{fb}} + \frac{p}{2} a_{0y}, \quad C_D = C_{D_{fb}} + \frac{p}{2} a_{0x}, \quad (6)$$

where subscript  $fb$  refers to the fixed body [12]. Here  $a_{0x}$  and  $a_{0y}$  are the dimensionless  $x$  and  $y$  components of oscillating flow acceleration. Since these accelerations are periodic their time-mean values vanish, resulting in identical TM values for the two setups. Equation (6) shows that for transverse motion the two drag coefficients are identical.

For the unheated case ( $T^*=1.0$ ) the present results are compared with the earlier data of the second author [13], who used his own code based on the finite difference method (FDM). The FDM code is set up for a mechanically-oscillated cylinder placed in a uniform stream, while the present FVM simulation is for oscillating flow around a stationary cylinder. However, when viewed from a system fixed to the cylinder, these two cases are kinematically identical (though dynamically not) and can thus be compared.

Figure 2 shows the TM value of drag (and fixed-body drag at the same time) against oscillation amplitude  $A_y$  for different temperature ratios. As seen in the figure, the TM of drag increases almost linearly with the oscillation amplitude and  $C_D$  also increases with increasing temperature ratio. For the isothermal case the present results agree well with the FDM data in [13].

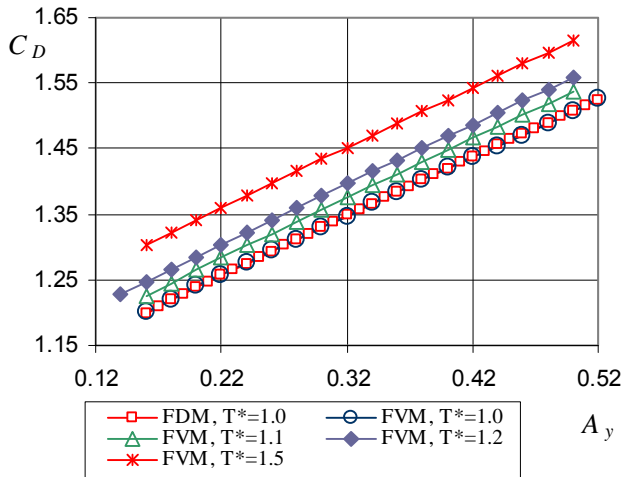


Fig. 2 Time-mean of drag versus amplitude  $A_y$  for different temperature ratios

The rms of drag is shown against oscillation amplitude for different temperature ratio values in Fig. 3. The results in [13] and those of the present work are almost identical for the isothermal case ( $T^*=0$ ).

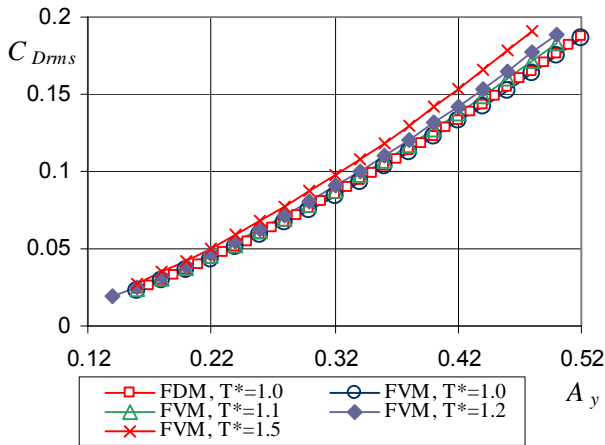


Fig. 3 Rms of drag versus amplitude  $A_y$  for different temperature ratios

Figure 4 shows the rms of fixed body lift against the oscillation amplitude for different temperature ratios. It can be seen that the rms of lift increases with increasing oscillation amplitude. As also seen in the figure, the lift coefficient decreases with increasing temperature ratio. For the unheated cylinder the results in [13] are also included in the figure for comparison, and the two sets of results compare very well.

For lift and drag coefficients it was observed that the wake behind the cylinder is not very sensitive to low temperature ratio values, while a significant effect is observable at  $T^*=1.5$ . For the uniform flow ( $A_y=0$ ) past a heated stationary cylinder it was found that the effect of heating is significant at the temperature ratio of  $T^*\geq 1.5$  [6], [14], [15].

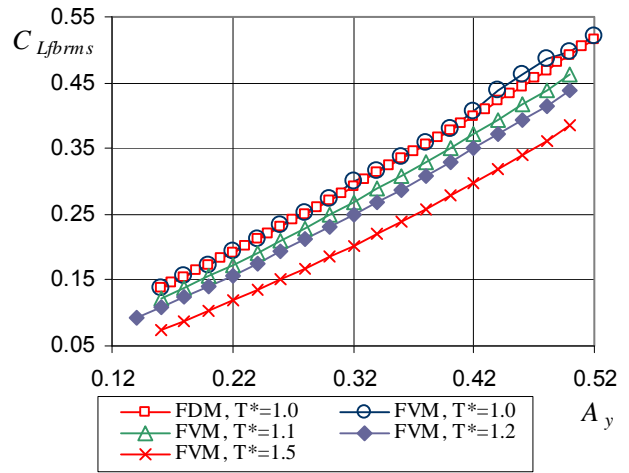


Fig. 4  $C_{Lfb\ rms}$  versus oscillation amplitude for different temperature ratios

### B. Heat transfer

The heat transfer between the cylinder and the surrounding fluid is calculated using the dimensionless heat transfer coefficient, or Nusselt number (see [16]),

$$Nu = \frac{hd}{k} = -\left(\frac{\partial T}{\partial r}\right)_{wall}, \quad (7)$$

where  $k$  (W/(m·K)) is the thermal conductivity of the fluid,  $h$  (W/(m<sup>2</sup>·K)) is the heat transfer coefficient and  $r$  is the dimensionless radius (non-dimensionalized by  $d$ ) and  $T$  is the dimensionless temperature defined by  $(\tilde{T} - \tilde{T}_\infty)/(\tilde{T}_w - \tilde{T}_\infty)$ , where  $\tilde{T}$  is the temperature of the fluid measured in K. In this study we investigate Nusselt number  $Nu_f = Nu(\tilde{T}_f)$  based on the film temperature  $\tilde{T}_f$

$$\tilde{T}_f = (\tilde{T}_w + \tilde{T}_\infty) / 2. \quad (8)$$

The uniform flow ( $A_y=0$ ) past a heated stationary cylinder was investigated first. The distribution of the local Nusselt number  $Nu_f$  over the cylinder surface is shown in Fig. 5 for  $T^*\approx 1.0$  obtained by FDM (see [16]) and the FVM results for  $T^*=1.1$  and  $T^*=1.5$ . Polar angle  $\theta=0^\circ$  at the front stagnation point where  $Nu_f$  is the largest due to the thin boundary layer. The local Nusselt number first decreases until it reaches a minimum value of about  $Nu_f=2$  near the separation point  $\theta_s$  and then slightly increases. At the separation point ( $\theta_s\approx 135^\circ$ ) the  $Nu$  values are roughly identical for all three cases.

The effect of temperature ratio on the local Nusselt number is investigated for oscillating cross flow and shown for the dimensionless amplitude value of 0.20 at the dimensionless instant of  $\tau = 218$  in Fig. 6. The local Nusselt number, just like for the stationary cylinder in uniform flow (see [15]), decreases with increasing temperature ratios for oscillating

flow, as seen in the figure. As an effect of heat transfer the value of the separation angle decreases slightly ( $\theta_s \approx 131.8^\circ$ ).

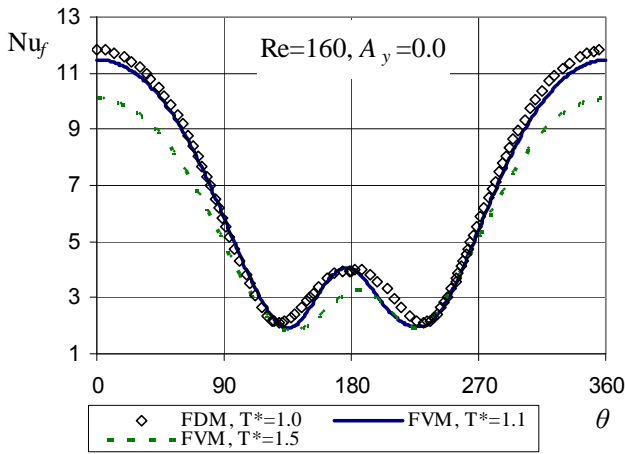


Fig. 5 Local distribution of  $Nu_f$  on the cylinder surface at  $Re=160$  and  $A_y=0$

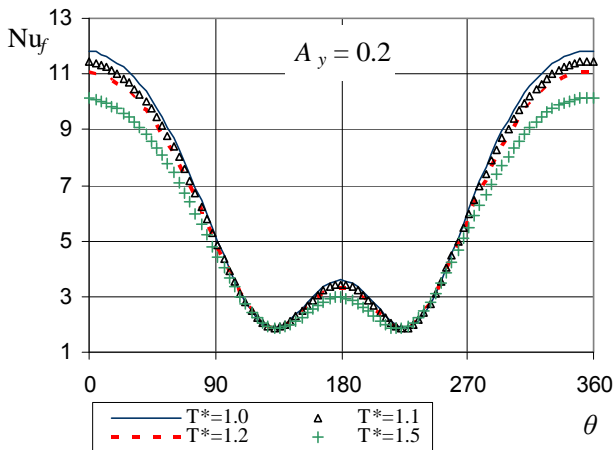


Fig. 6 Local distribution of  $Nu_f$  at  $A_y=0.20$  for different temperature ratios

In addition, the local Nusselt number was investigated at a given temperature ratio of 1.5 and amplitude of 0.20 for a complete cycle of vortex shedding at four different phases of vortex shedding. Figure 7 shows local Nusselt numbers at dimensionless times of  $\tau$ ;  $\tau+P/4$ ;  $\tau+P/2$ ;  $\tau+3P/4$ , where  $\tau = 218$  (the maximum velocity value of oscillation belongs to this dimensionless time) and  $P$  is a period of vortex shedding. The curves are similar in shape and magnitude, but a substantial discrepancy between Nusselt numbers at a fixed surface point can be found within a period of vortex shedding.

At the same dimensionless times the temperature contours of the wake in different phases of a vortex shedding cycle are shown for  $T^*=1.5$  and  $A_y=0.20$  in Fig. 8. This illustrates the effect of the phase of vortex shedding on the temperature contours in the wake of the cylinder.

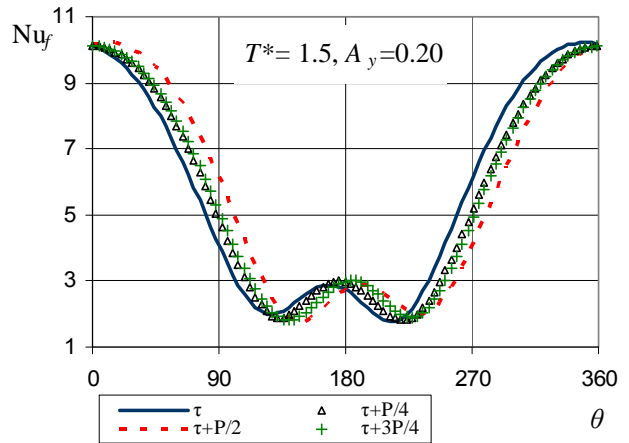


Fig. 7 The local distribution of  $Nu_f$  at  $T^*=1.5$  and  $A_y=0.2$  for a complete cycle of vortex shedding

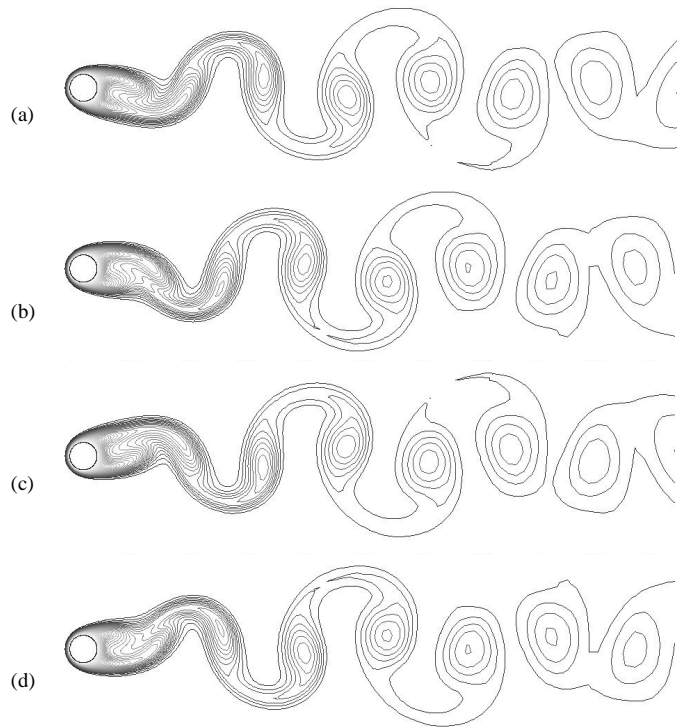


Fig. 8 Isothermal contours in different phases within a cycle (a)  $\tau$ ; (b)  $\tau+P/4$ ; (c)  $\tau+P/2$ ; (d)  $\tau+3P/4$

Integrating the local Nusselt number over the cylinder surface, the peripherally averaged Nusselt number is obtained, which is a periodic function of time in the locked-in region. The TM of this peripherally averaged Nusselt number yields the time-averaged Nusselt number. For free stream flow without an oscillating component around a heated stationary cylinder, the TM of  $Nu_f$  was analyzed at different Reynolds numbers in [15]. The agreement of the computational results of [15] with those of [14] is very good.

For transverse motion TM values of  $Nu_f$  are shown in Fig. 9

for different temperature ratios. The TM of Nusselt number increases with increasing oscillation amplitude, but decreases with increasing temperature, meaning that transverse flow or cylinder oscillation increases but the cylinder heating reduces the heat transfer per unit temperature difference between the fluid and cylinder. A similar tendency was found for a stationary cylinder placed in a uniform flow [14], [15].

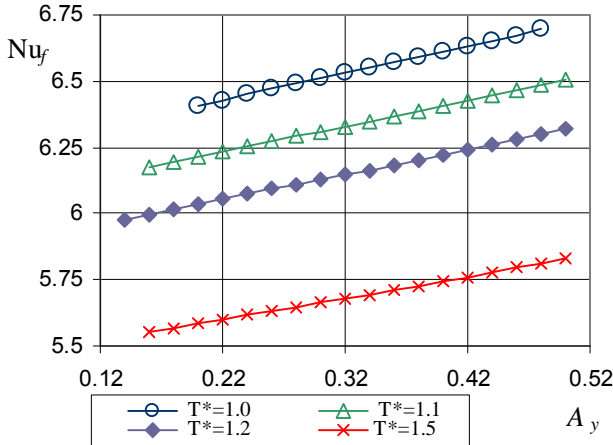


Fig. 9 Time-mean of  $Nu_f$  versus amplitude for different temperature ratios

C. Mechanical energy transfer

The mechanical energy transfer between the fluid and the cylinder for transverse cylinder motion was defined in [3]. For transverse motion the energy transfer coefficients can be written as follows:

$$E = \int_0^T C_L(t) \dot{y}_0(t) dt, \tag{9}$$

where  $y_0$  represents the dimensionless displacement of the flow in  $y$  direction. The over-dot means differentiation by dimensionless time.

Figure 10 shows the mechanical energy transfer for different temperature ratios. As can be seen in the figure, the energy transfer first increases with increasing amplitude, reaches a maximum value, and then decreases with increasing amplitude. The energy transfer is positive for the smaller amplitude values and negative for the larger  $A_y$  values. Positive  $E$  values mean that energy is added to the cylinder from the fluid, and so flow-induced vibration is liable to occur. By increasing the  $T^*$  value, the energy transfer curve shift to smaller  $E$  values. For the unheated cylinder ( $T^*=0$ ) the agreement between FDM [13] results and the present FVM results is very good.

Figure 11 shows the limit cycles ( $y_0, C_L$ ) for four different temperature ratios at  $A_y=0.20$ . The area enclosed by the limit cycles ( $y_0, C_L$ ) represents the mechanical energy transfer  $E$  (see [3], [17]). As can be seen in the figure, by increasing the amplitude, the shape of the curves varies.  $E$  is positive when

the orientation of the limit cycle curve is clockwise. Since this is true for all curves shown in Fig. 11,  $E$  is positive at  $A_y=0.20$ , as can also be seen in Fig. 10.

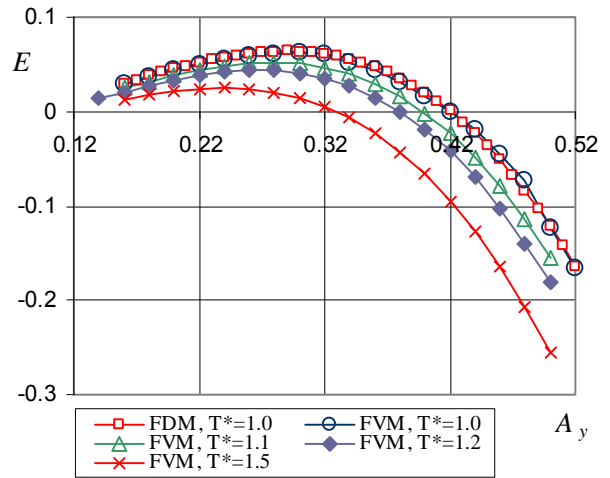


Fig. 10 Mechanical energy transfer versus amplitude for different temperature ratios

Limit cycle curves ( $y_0, C_L$ ) were determined also for a larger amplitude of 0.5 and for the  $T^*$  values shown earlier, and the curves are given in Fig. 12.  $E$  is negative when the orientation of the limit cycle curve is counterclockwise. It can be seen in that this holds for every curve in Fig. 12, meaning that  $E$  is negative. This harmonizes with the results shown in Fig. 10 at  $A_y=0.5$ .

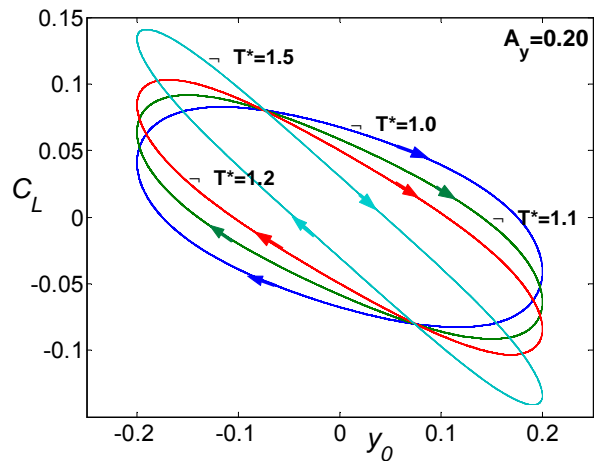


Fig. 11 Limit cycles ( $y_0, C_L$ ) at  $A_y=0.20$  for different temperature ratios

As can be seen in Figs. 11 and 12, at both amplitude values it is observed that all limit cycle curves go through the same two points on the ( $y_0, C_L$ ) plane. We have not yet found an explanation for this.

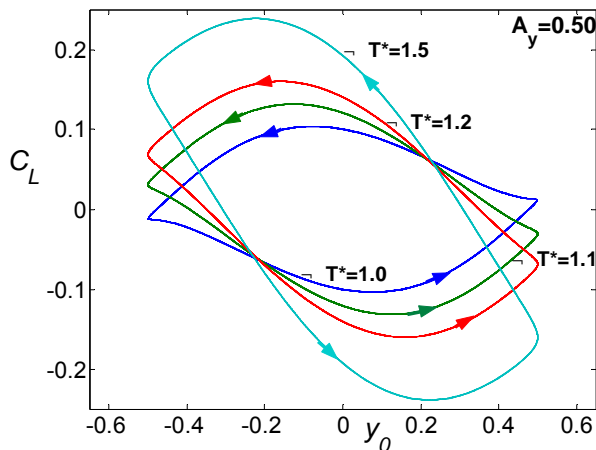


Fig. 12 Limit cycles ( $y_0$ ,  $C_L$ ) at  $A_y=0.50$  for different temperature ratios

#### IV. CONCLUSIONS

The influence of temperature ratio and oscillation amplitude was investigated on the two-dimensional superposition of uniform free stream flow and transversely oscillating flow of an incompressible fluid past a heated stationary circular cylinder. Investigations were based on the finite volume method and the dependence of fluid properties were taken into account.

In this study several computational aspects were investigated at a given Reynolds number of 160. For an unheated cylinder the present results were compared with those obtained by a finite difference method and good agreement was found.

With increasing surface temperature values the time-mean drag coefficients increase almost linearly, while the root-mean-square values of lift and time-mean of the Nusselt number decrease.

The distribution of the local Nusselt number over the cylinder surface was investigated over a complete vortex shedding cycle for a given temperature ratio and amplitude. It was found that the curves belonging to different phases are similar in shape and magnitude, but the Nusselt numbers at a fixed surface point can differ substantially.

Both positive (enhancing cylinder motion) and negative (damping cylinder motion) values occur in energy transfer for the superposition of uniform main stream and transverse oscillation, and the  $E$  values decrease at a given amplitude with increasing temperature ratio.

The wake is not very sensitive to low temperature ratios, while a significant effect was observed at  $T^*=1.5$ .

Future plans include the investigation of flow at other Reynolds numbers and frequency ratios.

#### REFERENCES

- [1] C.H.K. Williamson, "Vortex-Induced Vibrations," *Annual Review of Fluid Mechanics*, vol. 36, pp. 413-455, 2004.
- [2] C. Norberg, "Flow around a circular cylinder: aspects of fluctuating lift," *Journal of Fluids and Structures*, vol. 15, pp. 459-469, 2001.
- [3] H.M. Blackburn and R.D. Henderson, "A study of two-dimensional flow past an oscillating cylinder," *Journal of Fluid Mechanics* vol. 385, pp. 255-286, 1999.
- [4] J.C. Lecordier, L. Hamma, and P. Paranthoën, "The control of vortex shedding behind heated circular cylinder at low Reynolds numbers," *Experiments in Fluids*, vol. 10, pp. 224-229, 1991.
- [5] F. Dumouchel, J.C. Lecordier, and P. Paranthoën, "The effective Reynolds number of a heated cylinder," *Int. Journal of Heat Mass Transfer*, vol. 41, no. 12, pp. 1787-1794, 1998.
- [6] A.B. Wang, Z. Trávníček, and K.C. Chia, "On the relationship of effective Reynolds number and Strouhal number for the laminar vortex shedding of a heated circular cylinder," *Phys. Fluids*, vol. 12, no. 6, pp. 1401-1410, 2000.
- [7] X.Y. Lu and C. Dalton, "Calculation of the timing of vortex formation from an oscillating cylinder," *Journal of Fluids and Structures*, vol. 10, pp. 527-541, 1996.
- [8] W.H. Cheng, J.L. Hong, and W. Aung, "Numerical prediction of lock-on effect on convective heat transfer from a transversely oscillating circular cylinder," *Int. J. Heat Mass Transfer*, vol. 40, no. 8, pp. 1825-1834, 1997.
- [9] W.S. Fu and B.H. Tong, "Numerical investigation of heat transfer from a heated oscillating cylinder in a cross flow," *International Journal of Heat and Mass Transfer*, vol. 45, pp. 3033-3043, 2002.
- [10] B. Bolló, "Grid independence study for flow around a stationary circular cylinder," In *2010 Proc. microCAD 2010, International Computer Science Conference*, Miskolc, Hungary, pp.1-6.
- [11] B. Bolló and L. Baranyi, "Computation of low-Reynolds number flow around a stationary circular cylinder," *Proc. 7th International Conference on Mechanical Engineering*, Budapest, 2010, pp. 891-896.
- [12] L. Baranyi, "Lift and drag evaluation in translating and rotating non-inertial systems," *Journal of Fluids and Structures*, vol. 20(1), pp. 25-34, 2005.
- [13] L. Baranyi, B. Bolló, and L. Daróczy, "Simulation of low-Reynolds number flow around an oscillated cylinder using two computational methods," *ASME 2011 Pressure Vessels and Piping Division Conference PVP2011*, Baltimore, Maryland, 2011, Paper No. PVP2011-57554 (accepted for publication).
- [14] A.B. Wang, Z. Trávníček, "On the linear heat transfer correlation of a heated circular cylinder in laminar crossflow using a new representative temperature concept," *International Journal of Heat and Mass Transfer*, vol. 44, no. 24, pp. 4635-4647, 2001.
- [15] B. Bolló, "Low Reynolds number flow around and heat transfer from a heated circular cylinder," *Int. Review of Applied Sciences and Engineering*, Akadémiai Kiadó, vol. 1, no.1-2, pp.15-20, January 2010.
- [16] L. Baranyi, "Computation of unsteady momentum and heat transfer from a fixed circular cylinder in laminar flow," *Journal of Computational and Applied Mechanics*, vol. 4, no. 1, pp. 13-25, 2003.
- [17] L. Baranyi, "Numerical simulation of flow around an orbiting cylinder at different ellipticity values," *Journal of Fluids and Structures*, vol. 24, pp. 883-906, 2008.